RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2017

THIRD YEAR (BATCH 2015-18)

Date : 18/12/2017 **PHYSICS (Honours)**

Time : 11.00 am – 1.00 pm **Paper : V (Gr. C & D)** Full Marks : 50

(Use a separate Answer Book for each group)

Group – C

Answer **any three** questions: (3 × 10)

1. a) Show that a free electron can not absorb a photon and conserve both energy and momentum in the process. 2

   b) What is the main difference between the wave function $\Psi(x,t)$ representing a quantum particle and the function $E(x,t)$ representing the wave amplitude of a classical wave? 2

   c) A spectral line of wavelength 600 nm has a natural linewidth of 10^{-4}nm. Estimate the lifetime of the excited state from where the transition have taken place. 2

   d) Why are observables in Quantum Mechanics chosen to be represented by Hermitian Linear Operator? 2

   e) Show that if the wave function of a particle is real, its average momentum must be zero. 2

2. a) A particle is moving in a one-dimensional potential well defined by

   $$v(x) = 0 \quad \text{for } -a < x < a$$
   $$= -V_0 \quad \text{elsewhere}$$

   Obtain the energy eigenvalue equation. 5

   b) Consider a free particle in 1D with definite energy $E = \dfrac{\hbar^2 k^2}{2m}$.

   i) How many linearly independent states share this energy?

   ii) What symmetry guarantees this degeneracy? 1+1

   c) The wavelength of light emitted by a ruby laser is 694.3 nm. Assuming that the emission of a photon of this wavelength accompanies the transition of an electron from the $n = 2$ level to the $n = 1$ level of an infinite square well, compute the length (L) of the well. 3

3. a) Consider a beam of electrons travelling to the right along the x axis with energy E. The potential energy is $V = 0$ for $x < 0$ but at $x = 0$ there is a potential step and the potential energy increases to V_0 (> 0) for $x > 0$. Assuming that $E > V_0$ calculate the reflectance and transmission coefficients. 6

   b) A 10 eV electron is incident on a potential barrier of height 25 eV and width 1 nm.

   i) Calculate the order of magnitude of the probability that the electron will tunnel through the barrier.

   ii) Repeat the calculation for a width of 0·1 nm. 2+2

4 a) Evaluate the following commutator bracket: $\left[L_z, P^2\right]$ 2½

   b) Show that $\dfrac{d\langle L_Z \rangle}{dt} = 0$ for any spherically symmetric potential. 2½

   c) Consider a wave function (at $t = 0$)

   $$\Psi(x) = A\exp\left(-\frac{\sigma^2 x^2}{2}\right)\exp(ikx)$$

i) Find out the normalization constant. 1½

ii) Determine probability density and probability current. 1½+2

5. a) The angular part of the wavefunction for an electron bound in a Hydrogen atom is $\psi(\theta,\phi) = A\left(5\Upsilon_4^3 + \Upsilon_6^3 + \Upsilon_6^0\right)$ where $\Upsilon_i^m(\theta,\phi)$ are the normalized spherical harmonics.

i) What is the value of normalization constant A? 2

ii) What is the probability of finding the atom in a state with $m = 3$? 2

iii) What is the expectation value of angular momentum operator L^2 ? 2

b) The radial part of the ground state wave function of the Hydrogen atom is given by $R_{gs} = \left(\frac{1}{\pi a_0^3}\right)^{1/2} e^{-r/a_0}$ where the symbols have their usual meaning. Calculate the most probable value of the electron radius in the ground state. 4

Group – D

Answer **any two** questions: (2 × 10)

6. a) What is Larmor precession in atomic spectra? Define Larmor frequency? Show that Larmor frequency is equal to the frequency difference observed in the normal Zeeman effect between the displaced and undisplaced spectral lines. 1+3+2

b) How does the spectra of hydrogen-like atoms differ from that of pure hydrogen atom? 1

A muon (μ) from cosmic rays is trapped by a proton to form a hydrogen like atom. Given that a muon is approximately 200 times heavier than an electron. Determine the longest wavelength of the spectral line (in the analogue of the Lyman series) of such an atom. 3

7. a) What are Einstein A and B coefficients? Obtained a relation between them. 1+3

b) Show that two level laser system is not possible. 3

c) A laser beam of wavelength $\lambda = 6000\,\overset{0}{A}$, power 10 mW and angular spread 1.5 X 10^{-4} rad is focussed by a lens of focal length 10 cm. Find (i) radius (ii) power density and (iii) radiation pressure of the image. 3

8. a) Which of the following molecules will show rotational (microwave) and vibrational (infrared) spectrum in the respective part of the electromagnetic wave spectrum (i) H_2, (ii) HCN, (iii) CO_2, (iv) H_2O, (v) O_2, (vi) KCl. Explain your answer. 3+1

b) Write down the importance of studying Raman spectra of a molecule.

Qualitatively explain the relative order of intensities of Rayleigh line, Stokes line and anti-Stokes line. 2+2

c) A laser operating at 500 nm is used to excite a molecule. If the Stokes line is observed at 770 cm^{-1}. Find out the approximate positions of Stokes and anti-Stokes lines in nm. 2

9. a) What is Lande g-factor? Obtained an expression for it in terms of l, s and j. (symbols have usual meaning) 1+3

b) Write down Hund's rule for the determination of ground state configuration of an atom.

Determine the ground state configuration as well as all the energy levels arising from the two 2p electrons in the carbon atom. 3+3

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